

# Superconductivity in a two-dimensional superconductor with Rashba and Dresselhaus spin-orbit couplings

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We present a general model with both Rashba and Dresselhaus spin-orbit couplings to describe a two-dimensional noncentrosymmetric superconductor. The combined effects of the two spin-orbit couplings on superconductivity are investigated in the framework of mean-field theory. We find that the Rashba and Dresselhaus spin-orbit couplings result in similar effects on superconductivity if they are present solely in the system. Mixing of spin-singlet and triplet pairings in electron band is induced under the assumption that each quasiparticle band is p-wave paired. If the two types of spin-orbit couplings appear jointly, both the singlet and triplet pairings are weakened and decreased down to their minimum values in the equal-Rashba-Dresselhaus case.

## INTRODUCTION

Superconductivity in materials without inversion symmetry has attracted a lot of interests after the discovery of the heavy fermion noncentrosymmetric (NCS) superconductor CePt<sub>3</sub>Si [1–7]. Due to the lack of inversion symmetry, antisymmetric spin-orbit coupling (SOC) is introduced [2, 8]. There are two typical SOC's namely the so-called Rashba [9] and Dresselhaus [10] SOC's. The former is related to the microscopic structural inversion asymmetry and can be described by the form  $H_{\text{RSOC}} = \alpha(\sigma_y k_x - \sigma_x k_y)$  in a two-dimensional (2D) system [9, 11], while the latter arises due to the bulk inversion asymmetry in crystalline structures and the interface inversion asymmetry, with the linear form  $H_{\text{DSOC}} = \beta(\sigma_x k_x - \sigma_y k_y)$  in a 2D case [10, 12–14].  $\alpha, \beta$  are the coupling constants of Rashba and Dresselhaus terms, respectively. SOC is crucial for the novel properties in NCS superconductors [2].

In most previous studies, people focus on the effect of Rashba type SOC upon superconductivity [2–7, 15]. The Rashba SOC is reported to induce spin splitting and mixing of the spin-singlet and triplet pairings in a 2D superconducting system [16]. Both the spin-singlet and triplet pairings are found to be enhanced by Rashba SOC [5]. In addition to the Rashba type SOC, Dresselhaus SOC also contributes to the band splitting. Consequently, the similar effect on superconductivity is expected in the presence of Dresselhaus SOC. However, the details in this case are still unknown. Moreover, the combination of Rashba and Dresselhaus SOC's has been realized in semiconductor quantum wells [14, 17] and ultra-cold atoms [18, 19]. It is found that a lot of interesting physical phenomena appear in the presence of both Rashba and Dresselhaus couplings [20–22]. While in the NCS superconductors, the combined effect of Rashba and Dresselhaus SOC's on superconductivity remains open.

In this paper, we introduce a simple model to describe a 2D NCS superconducting system in the presence of both Rashba and Dresselhaus SOC's. Then the

combined effect of the two SOC's on superconductivity can be investigated in this model. The pairing order parameters are solved self-consistently within the mean-field theory. It is shown that an admixture of spin-singlet and triplet pairing can be induced by either pure Rashba/Dresselhaus SOC or the combination of the two SOC's. When Rashba and Dresselhaus SOC's are present solely, both the spin-singlet and triplet pairings are enhanced by increasing SOC. While in the case of Rashba and Dresselhaus SOC's are mixed, the two pairing gaps are weakened continuously with increasing Dresselhaus component and reduced to their minimum values in equal-Rashba-Dresselhaus case ( $\alpha = \beta$ ).

## THE MODEL

We start from the normal state Hamiltonian in the presence of both Rashba and Dresselhaus couplings as follows

$$H_N = \sum_{\mathbf{k}, s} \varepsilon_{\mathbf{k}} c_{\mathbf{k}s}^\dagger c_{\mathbf{k}s} + H_{\text{soc}}, \quad (1)$$

with

$$H_{\text{soc}} = \sum_{\mathbf{k}, ss'} \{ \alpha(\sigma_y k_x - \sigma_x k_y) + \beta(\sigma_x k_x - \sigma_y k_y) \}_{ss'} c_{\mathbf{k}s}^\dagger c_{\mathbf{k}s'}, \quad (2)$$

where  $\varepsilon_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m} - \mu$  is the spin-independent single electron kinetic energy measured relative to the chemical potential  $\mu$ .  $c_{\mathbf{k}s}^\dagger (c_{\mathbf{k}s'})$  is the creation (annihilation) operator of electron and  $s, s' = \uparrow, \downarrow$  are spin indices.  $\alpha$  and  $\beta$  are the Rashba and Dresselhaus SOC strength parameters, respectively.  $\mathbf{k} = (k_x, k_y)$  is the 2D electron wave vector, and  $\sigma_x, \sigma_y$  are the Pauli matrices.

By introducing an angle  $\theta$  which denotes the strength ratio between Rashba and Dresselhaus SOC's [12, 23], the Eq. (2) can be rewritten as

$$H_{\text{soc}} = \sum_{\mathbf{k}, ss'} \gamma (\tilde{\sigma}_y k_x - \tilde{\sigma}_x k_y)_{ss'} c_{\mathbf{k}s}^\dagger c_{\mathbf{k}s'}, \quad (3)$$

with  $\gamma = \sqrt{\alpha^2 + \beta^2}$ , and  $\alpha = \gamma \cos \theta$ ,  $\beta = \gamma \sin \theta$ ,  $\tilde{\sigma}_x = \begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix}$ , and  $\tilde{\sigma}_y = \begin{pmatrix} 0 & -ie^{i\theta} \\ ie^{-i\theta} & 0 \end{pmatrix}$ . Then the Hamiltonian in Eq. (1) reads

$$H_N = \sum_{\mathbf{k}, s} \varepsilon_{\mathbf{k}} c_{\mathbf{k}s}^\dagger c_{\mathbf{k}s} + \sum_{\mathbf{k}, ss'} \gamma (\tilde{\sigma}_y k_x - \tilde{\sigma}_x k_y)_{ss'} c_{\mathbf{k}s}^\dagger c_{\mathbf{k}s'}. \quad (4)$$

Applying the diagonalization procedure with Bogliubov transformation in Eq. (4), we arrive at

$$H_N = \sum_{\mathbf{k}\lambda} \xi_{\mathbf{k}}(\mathbf{k}) a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda}, \quad (5)$$

where  $a_{\mathbf{k}\lambda}^\dagger (a_{\mathbf{k}\lambda})$  is the creation (annihilation) operator of quasiparticle, and  $\lambda = \pm$  labels the SOC lifted quasiparticle band.  $\xi_{\mathbf{k}}(\mathbf{k}) = \varepsilon_{\mathbf{k}} - \lambda \gamma |\mathbf{k}| \varsigma(\theta, \phi_{\mathbf{k}})$  is the energy dispersion in each quasiparticle band with  $\varsigma(\theta, \phi_{\mathbf{k}}) = \sqrt{1 - \sin 2\theta \sin 2\phi_{\mathbf{k}}}$  and  $|\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$ . Also we get the following unitary transformations

$$\begin{aligned} C_{\mathbf{k}\uparrow} &= \frac{1}{\sqrt{2}} a_{\mathbf{k}+} + \frac{1}{\sqrt{2}} e^{i\eta(\mathbf{k}, \theta)} a_{\mathbf{k}-}, \\ C_{\mathbf{k}\downarrow} &= \frac{1}{\sqrt{2}} a_{\mathbf{k}-} - \frac{1}{\sqrt{2}} e^{-i\eta(\mathbf{k}, \theta)} a_{\mathbf{k}+}, \end{aligned} \quad (6)$$

with  $e^{i\lambda\eta(\mathbf{k}, \theta)} = \frac{i\lambda e^{i\lambda\theta} \sin \phi_{\mathbf{k}} + e^{-i\lambda\theta} \cos \phi_{\mathbf{k}}}{\varsigma(\theta, \phi_{\mathbf{k}})}$ , and  $\tan \phi_{\mathbf{k}} = k_x/k_y$ .

At zero temperature, the two quasiparticle bands are filled up to the same Fermi energy level  $\epsilon_F$ , but with different Fermi wave vectors. There are two Fermi contour lines corresponding to two different dispersions  $\xi_{\mathbf{k}}(\mathbf{k})$  as shown in Fig. 1. For the system displaying pure Rashba ( $\theta = 0$ ) or pure Dresselhaus ( $\theta = \pi/2$ ) SOC, the Fermi contour lines show similar isotropic concentric circles [see Fig. 1(a)]. Rashba and Dresselhaus terms are found to play different roles on the spin orientations in  $\mathbf{k}$ -space [13, 14], however it is not considered in this paper. In the presence of both Rashba and Dresselhaus couplings, the two Fermi contour lines are anisotropic and non-equivalent along  $[110]$  and  $[\bar{1}10]$  directions as plotted in Figs. 1(b) and 1(c). Especially, when  $\alpha = \beta$ , the two Fermi contour lines touch at  $[110]$  direction, displayed in Fig. 1(c). It is shown that spin-splitting vanishes along a certain direction in this case [24], and nontrivial physical properties can be expected.

In the strong SOC limit [6, 7],  $k_B T_C \ll \gamma < \mu$ , the theory of NCS superconductor is analogous to that of ferromagnetic superconductors [25], and then only intra-band pairing is allowed to occur in the same quasiparticle band [5–7]. Consequently, a p-wave pairing is considered to be occur in each quasiparticle band, and the pairing Hamiltonian can be written as

$$H_{sc} = \frac{1}{2N} \sum_{\mathbf{k}, \mathbf{k}', \lambda} V_{\lambda}(\mathbf{k}, \mathbf{k}') a_{\mathbf{k}\lambda}^\dagger a_{-\mathbf{k}\lambda}^\dagger a_{-\mathbf{k}'\lambda} a_{\mathbf{k}'\lambda}, \quad (7)$$

where  $N$  is the number of  $\mathbf{k}$  points. We set the pairing potential  $V_{\lambda}(\mathbf{k}, \mathbf{k}') = -V_{\lambda}(-\mathbf{k}, \mathbf{k}') = -V_{\lambda}(\mathbf{k}, -\mathbf{k}') =$

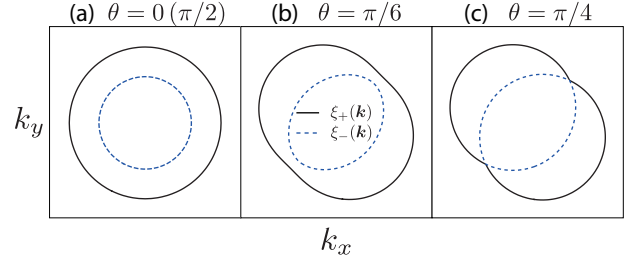


FIG. 1: (Color online) The Fermi contour in the presence of Rashba and Dresselhaus SOC with different  $\theta$ . (a)  $\theta = 0(\pi/2)$  represents only Rashba or Dresselhaus SOC. (b)  $\theta = \pi/6$  stands for the combination of Rashba and Dresselhaus ones and  $\alpha > \beta$ . (c)  $\theta = \pi/4$  shows the equal-Rashba-Dresselhaus case,  $\alpha = \beta$ .

$-V e^{i\lambda(\eta(\mathbf{k}, \theta) - \eta(\mathbf{k}', \theta))}$  as analyzed in previous studies [5, 26], and  $\Delta_{\lambda}(\mathbf{k}) = -\lambda \Delta_{\lambda} e^{i\lambda\eta(\mathbf{k}, \theta)}$ . In the weak coupling approach, the pairing interaction is nonzero only inside the thin shells of width  $\omega_c$  in the vicinity of Fermi surface, and  $\omega_c$  is chosen to be same in each quasiparticle band.

To obtain the pairing order parameters, we define Green's functions  $G_{\lambda}(\mathbf{k}, \tau - \tau') = -\langle T_{\tau} a_{\lambda}(\mathbf{k}, \tau) a_{\lambda}^\dagger(\mathbf{k}, \tau') \rangle$ ,  $F_{\lambda}(\mathbf{k}, \tau - \tau') = \langle T_{\tau} a_{\lambda}(\mathbf{k}, \tau) a_{\lambda}(-\mathbf{k}, \tau') \rangle$ . The motion equations of Green's functions in each band can be written as follows

$$\begin{aligned} \{i\omega_n - \xi_{\lambda}(\mathbf{k})\} G_{\lambda}(\mathbf{k}, \omega_n) + \Delta_{\lambda}(\mathbf{k}) F_{\lambda}^\dagger(-\mathbf{k}, \omega_n) &= 1, \\ \{i\omega_n + \xi_{\lambda}(\mathbf{k})\} F_{\lambda}^\dagger(-\mathbf{k}, \omega_n) + \Delta_{\lambda}^\dagger(\mathbf{k}) G_{\lambda}(\mathbf{k}, \omega_n) &= 0. \end{aligned} \quad (8)$$

Then the obtained Green's functions read

$$G_{\lambda}(\mathbf{k}, \omega_n) = \frac{i\omega_n + \xi_{\lambda}(\mathbf{k})}{(i\omega_n)^2 - E_{\lambda}^2(\mathbf{k})}, \quad (9)$$

$$F_{\lambda}^\dagger(\mathbf{k}, \omega_n) = \frac{-\Delta_{\lambda}^*(\mathbf{k})}{(i\omega_n)^2 - E_{\lambda}^2(\mathbf{k})}, \quad (10)$$

where  $E_{\lambda}(\mathbf{k}) = \sqrt{\xi_{\lambda}^2(\mathbf{k}) + \Delta_{\lambda}^2(\mathbf{k})}$  is the quasiparticle excitation energy for each band. The pairing order parameter in each quasiparticle band is defined as

$$\Delta_{\lambda}(\mathbf{k}) = -\frac{1}{N} \sum_{\mathbf{k}'} V_{\lambda}(\mathbf{k}\mathbf{k}') F_{\lambda}(\mathbf{k}', 0). \quad (11)$$

The chemical potential  $\mu$  is determined from the particle number density

$$n = \frac{1}{N} \sum_{\mathbf{k}} (\langle n_{\mathbf{k}\uparrow} \rangle + \langle n_{\mathbf{k}\downarrow} \rangle). \quad (12)$$

With  $\langle n_{k\lambda} \rangle = \frac{1}{2} - \frac{\xi_{\lambda}(\mathbf{k})}{2E_{\lambda}(\mathbf{k})} \tanh \frac{E_{\lambda}(\mathbf{k})}{2k_B T}$ , the order parameters equations should satisfy

$$\Delta_{\lambda}(\mathbf{k}) = -\frac{1}{N} \sum_{\mathbf{k}'} V_{\lambda}(\mathbf{k}\mathbf{k}') \frac{\Delta_{\lambda}(\mathbf{k}') \tanh \frac{E_{\lambda}(\mathbf{k}')}{2k_B T}}{2E_{\lambda}(\mathbf{k}')}, \quad (13)$$

$$n = \frac{1}{N} \sum_{\mathbf{k}} \left\{ 1 - \frac{\xi_+(\mathbf{k})}{2E_+(\mathbf{k})} \tanh \frac{E_+(\mathbf{k})}{2k_B T} - \frac{\xi_-(\mathbf{k})}{2E_-(\mathbf{k})} \tanh \frac{E_-(\mathbf{k})}{2k_B T} \right\}. \quad (14)$$

In the 2D case, the summations over  $\mathbf{k}$  space in Eqs. (13) and (14) can be converted into continuum integrals over energy by  $\sum_{\mathbf{k}} = \frac{N}{(2\pi)^2} \int d^2\mathbf{k} = \frac{N}{(2\pi)^2} \int k dk d\varphi$ , where  $k$  is the magnitude of the momentum  $\mathbf{k}$  and  $\varphi$  is the polar angle. The unit of the energy can be scaled by the factor  $\frac{\hbar^2(2\pi n)}{2m}$  and the particle number density is set as  $n = 1$  for half-filling. Accordingly, Eqs. (13) and (14) can be rewritten in the zero temperature limit with  $\tanh \frac{E_\lambda(\mathbf{k})}{2k_B T} \rightarrow 1$  as following form

$$\bar{\Delta}_\lambda = \frac{\bar{V}}{8\pi} \int_{\bar{\epsilon}_\lambda - \bar{\omega}_c}^{\bar{\epsilon}_\lambda + \bar{\omega}_c} d\bar{\epsilon} \int_0^{2\pi} d\varphi \frac{\bar{\Delta}_\lambda}{\bar{E}_\lambda}, \quad (15)$$

$$1 = \frac{1}{8\pi} \int_0^\infty d\bar{\epsilon} \int_0^{2\pi} d\varphi \left( 2 - \frac{\bar{\xi}_+}{\bar{E}_+} - \frac{\bar{\xi}_-}{\bar{E}_-} \right), \quad (16)$$

where  $\bar{E}_\lambda = \sqrt{\bar{\xi}_\lambda^2 + \bar{\Delta}_\lambda^2}$ , and  $\bar{\xi}_\lambda = \bar{\epsilon} - \bar{\mu} - \lambda \bar{\gamma}_{\text{soc}} \varsigma(\theta, \varphi) \sqrt{\bar{\epsilon}}$  with  $\varsigma(\theta, \varphi) = \sqrt{1 - \sin 2\theta \sin 2\varphi}$ , and  $\varphi = \frac{\pi}{2} - \phi_{\mathbf{k}}$ . The combined SOC's strength is scaled as  $\bar{\gamma}_{\text{soc}} = \sqrt{2\bar{\gamma}^2 m}$ , and  $\bar{\epsilon}_\lambda = \frac{1}{2}(\bar{\gamma}_{\text{soc}}^2 \varsigma^2(\theta, \varphi) + 2\bar{\mu} + \lambda \sqrt{4\bar{\gamma}_{\text{soc}}^2 \mu \varsigma^2(\theta, \varphi) + \bar{\gamma}_{\text{soc}}^4 \varsigma^4(\theta, \varphi)})$ . The dimensionless pairing potential  $\bar{V}$  is defined as  $\bar{V} = V * (\frac{\hbar^2(2\pi n)}{2m})^{-1}$ , and the dimensionless energies  $\bar{\epsilon}$ ,  $\bar{\mu}$ ,  $\bar{\Delta}_\lambda$  and  $\bar{\gamma}_{\text{soc}}$  are defined analogously. The energy cutoff is chosen as  $\bar{\omega}_c = 0.01$ .

When transformed into electron space using the unitary transformations Eq. (6), the pairing Hamiltonian Eq. (7) is an admixture of s-wave and p-wave pairing terms,

$$H_{sc} = \frac{1}{4N} \sum_{\mathbf{k}, \mathbf{k}', \sigma} V_\sigma(\mathbf{k}, \mathbf{k}') c_{\mathbf{k}\sigma}^\dagger c_{-\mathbf{k}\sigma}^\dagger c_{-\mathbf{k}'\sigma} c_{\mathbf{k}'\sigma} - \frac{1}{2N} \sum_{\mathbf{k}, \mathbf{k}', \sigma} V c_{\mathbf{k}\sigma}^\dagger c_{-\mathbf{k}, -\sigma}^\dagger c_{-\mathbf{k}', -\sigma} c_{\mathbf{k}', \sigma}, \quad (17)$$

where  $V_\sigma(\mathbf{k}, \mathbf{k}') = -V e^{i\sigma(\eta(\mathbf{k}, \theta) - \eta(\mathbf{k}', \theta))}$  is the electron p-wave pairing potential and  $\sigma = \pm 1$  represents the electron spin. The superconducting order parameters in electron space are also an admixture of spin-singlet and spin-triplet ones accordingly, and can be expressed as [5]

$$\begin{aligned} \Delta_{\uparrow\uparrow}(\mathbf{k}) &= -\frac{e^{i\eta(\mathbf{k}, \theta)}}{2} (\bar{\Delta}_+ + \bar{\Delta}_-), \\ \Delta_{\downarrow\downarrow}(\mathbf{k}) &= \frac{e^{-i\eta(\mathbf{k}, \theta)}}{2} (\bar{\Delta}_+ + \bar{\Delta}_-), \\ \Delta_{\uparrow\downarrow} &= \frac{1}{2} (\bar{\Delta}_+ - \bar{\Delta}_-). \end{aligned} \quad (18)$$

From the equations above, we can calculate the effect of Rashba and Dresselhaus SOC's on electron superconducting order parameters in both spin-singlet and triplet channels.

## RESULTS AND DISCUSSIONS

Firstly, we can calculate the quasiparticle pairing gaps  $\bar{\Delta}_+$  and  $\bar{\Delta}_-$  self-consistently by solving Eqs. (15) and (16). As shown in Fig. 2(a), no matter Rashba and Dresselhaus SOC's are present solely ( $\theta = 0, (\pi/2)$ ) or jointly ( $\theta = \pi/6, \pi/4$ ),  $\bar{\Delta}_+$  is always enhanced by SOC, while  $\bar{\Delta}_-$  is weakened as SOC increased displaying in Fig. 2(b). The role of SOC in quasiparticle pairings analogous to that of the ferromagnetism in ferromagnetic superconductors [25]. When Rashba and Dresselhaus SOC's are mixed ( $\theta = \pi/6, \pi/4$ ),  $\bar{\Delta}_+$  is reduced with the increase of Dresselhaus component, while  $\bar{\Delta}_-$  is strengthened. The mixed SOC seems to soften the effect of pure Rashba or Dresselhaus SOC on the quasiparticle pairings.

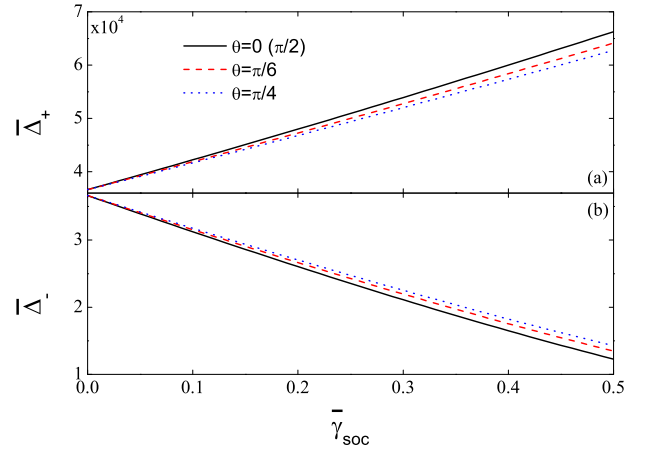


FIG. 2: (Color online) The dependencies of the quasiparticle superconducting order parameters  $\bar{\Delta}_+$  (a),  $\bar{\Delta}_-$  (b) on SOC strength  $\bar{\gamma}_{\text{soc}}$  are plotted at  $\bar{T} = 0$  and  $\bar{V} = 0.5$ . The angle  $\theta = 0, \pi/6, \pi/4$  represent  $\beta = 0, \alpha > \beta$  and  $\alpha = \beta$ , respectively.

With  $\bar{\Delta}_+$  and  $\bar{\Delta}_-$  in hand and according to Eq. (18), we can investigate the electron pairings upon increasing SOC. From the Eq. (18), it is clearly shown that  $|\Delta_{\uparrow\uparrow}(\mathbf{k})| = |\Delta_{\downarrow\downarrow}(\mathbf{k})|$ . For convenience, we set  $\bar{\Delta}_{\uparrow\uparrow} = |\Delta_{\uparrow\uparrow}(\mathbf{k})| + |\Delta_{\downarrow\downarrow}(\mathbf{k})|$ ,  $\bar{\Delta}_{\uparrow\downarrow}$  as the electron spin-triplet and singlet pairing components, respectively. In the absence of SOC,  $\bar{\Delta}_+ = \bar{\Delta}_-$ , the spin-singlet pairing component  $\bar{\Delta}_{\uparrow\downarrow} = 0$ , i.e., the superconducting state is a pure triplet one.

The order parameters  $\bar{\Delta}_{\uparrow\uparrow}$ ,  $\bar{\Delta}_{\uparrow\downarrow}$  and the ratio  $R_{\bar{\Delta}} = \bar{\Delta}_{\uparrow\downarrow}/\bar{\Delta}_{\uparrow\uparrow}$  are plotted as a function of  $\bar{\gamma}_{\text{soc}}$  in Fig. 3. When pure Rashba or Dresselhaus SOC is present ( $\theta = 0, (\pi/2)$ ), both the spin-triplet and singlet pairing components are enhanced with increasing SOC. The only difference between Rashba and Dresselhaus SOC's on the pairing gaps is related to a phase factor as shown in Eq. (18). For the pure Rashba SOC system,  $e^{i\lambda\eta(\mathbf{k}, \theta)} = e^{i\lambda\phi_{\mathbf{k}}}$ , while in the pure Dresselhaus SOC case  $e^{i\lambda\eta(\mathbf{k}, \theta)} = -i\lambda e^{-i\lambda\phi_{\mathbf{k}}}$ . However, in this paper we only focus on the magnitude of the order parameter. In this case, the pure Rashba and Dresselhaus SOC's are considered to result in the similar effect on the pairing gaps. In the presence of

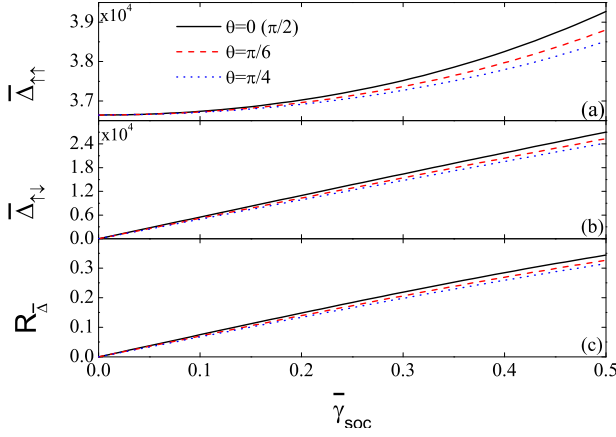


FIG. 3: (Color online) Plots of the electron pairing order parameters spin-triplet component  $\bar{\Delta}_{\uparrow\uparrow}$  (a), spin-singlet component  $\bar{\Delta}_{\uparrow\downarrow}$  (b) and the ratio of spin-singlet to spin-triplet component  $R_{\bar{\Delta}}$  (c) as functions of  $\bar{\gamma}_{soc}$  with different  $\theta$ . All the parameters are solved at  $\bar{T} = 0$  and  $\bar{V} = 0.5$ .

both Rashba and Dresselhaus SOC ( $\theta = \pi/6, \pi/4$ ), the combined SOC also enhance the spin-singlet and triplet pairings. Moreover, for a fixed  $\bar{\gamma}_{soc}$ , the electron pairings in both spin-triplet and spin-singlet channels are weakened by increasing Dresselhaus component (see Figs. 3(a) and 3(b)). As displayed in Fig. 3(c), the ratio  $R_{\bar{\Delta}}$  increases from zero as a function of  $\bar{\gamma}_{soc}$  in all the cases  $\theta = 0, (\pi/2)$ ,  $\theta = \pi/6$  and  $\theta = \pi/4$ , implying that spin-singlet pairing is induced and increased by pure Rashba or Dresselhaus SOC and the mixed SOC.

Fig. 4 displays the pairing parameters  $\bar{\Delta}_{\uparrow\uparrow}$ ,  $\bar{\Delta}_{\uparrow\downarrow}$  and the ratio  $R_{\bar{\Delta}}$  as the variation of  $\theta$ . As  $\theta$  increases, the Dresselhaus component in the combined SOC is increased. All the  $\bar{\Delta}_{\uparrow\uparrow}$ ,  $\bar{\Delta}_{\uparrow\downarrow}$  and  $R_{\bar{\Delta}}$  are shown to reduce continuously with increasing  $\theta$ , and decrease down to their minimum values right at  $\theta = \pi/4$  ( $\alpha = \beta$ ). In the case of  $\alpha = \beta$ , the splitting of the Fermi contour lines vanishes at a certain direction as shown in Fig. 1(c). The minimum values may correspond to this splitting vanishing. As  $\theta$  increases further, the order parameters and the ratio  $R_{\bar{\Delta}}$  rise again. When  $\theta$  increases to  $\theta = \pi/2$  ( $\alpha = 0$ ), only Dresselhaus SOC establishes in the system, and the values of all the order parameters are maximal and equal to those when  $\theta = 0$ .

## SUMMARY

In summary, the combined effect of the Rashba and Dresselhaus SOC on superconductivity is investigated within the mean-field theory. Either the Rashba or Dresselhaus SOC can mix the spin degree of freedom and thus may give rise to two nondegenerate quasiparticle bands. This explains why spin-singlet pairing in the electron band can be induced from the spin-triplet pairing of quasiparticles. Both spin-singlet and triplet electron-pairing are enhanced with the strength of SOC increased.

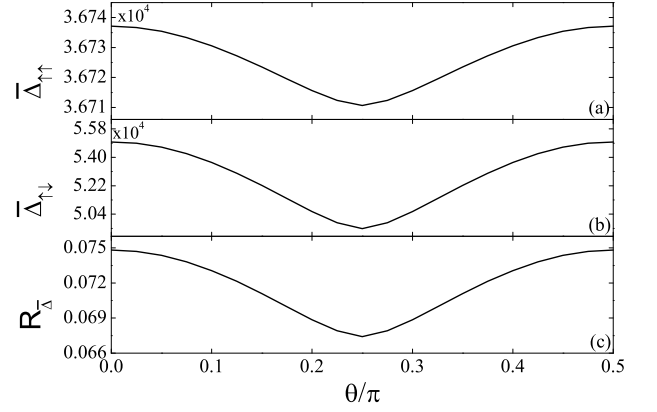


FIG. 4: Shown are the plots of the electron pairing gaps  $\bar{\Delta}_{\uparrow\downarrow}$  (a),  $\bar{\Delta}_{\uparrow\uparrow}$  (b) and the ratio  $R_{\bar{\Delta}}$  (c) as functions of  $\theta$  at  $\bar{T} = 0$ ,  $\bar{V} = 0.5$  and  $\bar{\gamma}_{soc} = 0.1$ .

However, if the two SOC are combined but the effective coupling strength keeps a constant, we find that both spin-singlet and triplet electron-pairing are weakened and decrease down to their minimum values in the case that the Rashba and Dresselhaus SOC are equally mixed. In this case, the Fermi contour lines of the two quasiparticle bands touch at a certain direction.

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- [1] E. Bauer, G. Hilscher, H. Michor, Ch. Paul, E. W. Scheidt, A. Gribanov, Yu. Seropegin, H. Noel, M. Sigrist and P. Rogl, Phys. Rev. Lett. **92** (2004) 027003.
- [2] P. A. Frigeri, D. F. Agterberg, A. Koga and M. Sigrist, Phys. Rev. Lett. **92** (2004) 097001; Erratum **93** (2004) 099903(E).
- [3] B. Liu and I. Eremin, Phys. Rev. B **78** (2008) 014518.
- [4] B. Liu and X. Hu, Phys. Rev. B **81** (2010) 144504; B. Liu, F. Yuan and X. Hu, J. Phys. Chem. Solids **72** (2011) 380.
- [5] J. Linder, A. Nevidomskyy, A. Sudbø, Phys. Rev. B **78** (2008) 172502.
- [6] K. V. Samokhin, E. S. Zijlstra and S. K. Bose, Phys. Rev. B **69** (2004) 094514; K. V. Samokhin and V. P. Mineev, Phys. Rev. B **77** (2008) 104520.

- [7] V. P. Mineev, M. Sigrist, Non-Centrosymmetric Superconductors, Lecture Notes in Physics, Vol. 847, edited by E. Bauer and M. Sigrist (Springer-Verlag, Berlin, 2012) 129-154.
- [8] R. Winkler, Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems, Springer, Berlin, 2003.
- [9] E. I. Rashba, Sov. Phys. Solid State **2** (1960) 1109.
- [10] G. Dresselhaus, Phys. Rev. **100** (1955) 580.
- [11] Y. A. Bychkov and E. I. Rashba, J. Phys. Chem. **17** (1984) 6039.
- [12] K. A. Vardanyan, A. L. Vartanian and A. A. Kirakosyan, Eur. Phys. J. B **85** (2012) 367.
- [13] S. D. Ganichev and W. Prettl, J. Phys.: Condens. Matter **15** (2003) R935.
- [14] S. D. Ganichev, V. V. Belkov, Leonid E. Golub, E. L. Ivchenko, Petra Schneider, S. Giglberger, J. Eroms, J. De Boeck, G. Borghs, W. Wegscheider, D. Weiss, and W. Prettl, Phys. Rev. Lett. **92** (2004) 256601.
- [15] X. Yan and Q. Gu, Physica C **493** (2013) 125-127.
- [16] L. P. Gor'kov and E. I. Rashba, Phys. Rev. Lett. **87** (2001) 037004.
- [17] J. D. Koralek, C. P. Weber, J. Orenstein, B. A. Bernevig, Shou-Cheng Zhang, S. Mack and D. D. Awschalom, Nature **458** (2009) 610.
- [18] Y. J. Lin, K. Jimenez-Garcia, and I. B. Spielman, Nature **471** (2011) 83.
- [19] V. Galitski and I. B. Spielman, Nature **494** (2013) 49.
- [20] Chao Li and Feng Zhai, J. Appl. Phys. **109** (2011) 093306.
- [21] L. Dell'Anna, G. Mazzeella and L. Salasnich, Phys. Rev. A **86** (2012) 053632.
- [22] Z. Li, L. Covaci and F. Marsiglio, Phys. Rev. B **85** (2012) 205112.
- [23] Z. H. Yang, Y. H. Yang, J. Wang, and K. S. Chan, J. Appl. Phys. **103** (2008) 103905.
- [24] C. Lopez-Bastidas, J. A. Maytorena and F. Mireles, Phys. Status Solidi C **4** (2007) 4229-4235.
- [25] X. L. Jian, J. C. Zhang, Q. Gu, R. A. Klemm, Phys. Rev. B **80** (2009) 224514.
- [26] J. Linder and A. Sudbø, Phys. Rev. B **76** (2007) 054511.